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B.A./B.Sc. V Semester Examination

$CBS - Vs/1(R\&P)$

Mathematics
Course No.: UMTTE503

Time Allowed: 3 Hours **Maximum Marks: 80**

Note: Attempt any four questions from the given questions. Each question carries equal marks.

Q 1. *i) Define vector subspace with two examples. Prove that a non-empty subset* W *of a vector space* V *over a field* F *is a vector subspace if and only if* $\alpha x + \beta y \in W$ *for all* $x,\ y \in W$ *and* $\alpha,\ \beta \in F$.

ii) Define vector space. Show that $\mathbb{C}$, *the complex plane is a vector space over* $\mathbb{Q}$.

Q 2. *i) Prove that intersection of any number of subspaces of a vector space* V *over a field* F *is a subspace of* V.

ii) Let W_1 *and* W_2 *be two subspaces of a vector space* V *over a field* F. *Prove that* $W_1 + W_2 = L(W_1 \cup W_2)$.

Q 3. *Define finitely generated vector space with an example. Prove that there exists a basis for each finitely generated vector space.*

Q 4. *i) Discuss linear dependence or independence of the following sets:*

(a) $\{xsinx,\ cosx\}$

(b) $\{(1,\ 1,\ 1,\ 1),\ (1,\ 1,\ 1,\ 0),\ (0,\ 1,\ 0,\ 1),\ (0,\ 1,\ 0,\ 0)\}$

ii) Let V *be a vector space over a field* F. *Show that the set* S *of non-zero vectors* $x_1,\ x_2,\ x_3,\ x_4 \in V$ *is L.D. if and only if some element of* S *is a linear combination of the others.*

Q 5. *i) Let V be a finite dimensional vector space over a field F. If W is a subspace of V, prove that dim.$W \leq$ dim.V.*

ii) Show that $\mathbb{R}$ is infinite dimensional vector space over $\mathbb{Q}$.

Q 6. *i) Let V and W be two vector spaces over the same field F. Show that $T: V \to W$ is a linear transformation if and only if $T(\lambda x + y) = \lambda T(x) + T(y)$ for all $x,\ y \in V$ and $\lambda \in F$.*

ii) Let W be subspace of $\mathbb{R}^3$ generated by $\{(1,\ 0,\ 0),\ (0,\ 0,\ 1)\}$. Find the quotient space $\mathbb{R}^3/W$ and its basis.

Q 7. *State and prove Rank-Nullity theorem.*

Q 8. *i) Find the matrix representation of $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as*

$$T(x,\ y) = (3x - 4y,\ x + 5y)$$

with respect to the basis $B = \{(1,\ 0),\ (0,\ 1)\}$.

ii) If A and B are symmetric matrices of the same order, then show that AB is symmetric if and only if $AB = BA$.